Investment and financing decisions under constrained demand*

Paulo J. Pereira[a] and Artur Rodrigues[b]

[a] *CEF.UP and Faculdade de Economia, Universidade do Porto, Portugal.*
[b] *NIPE and School of Economics and Management, University of Minho, Portugal.*

January 2024
Early draft

Abstract

This paper examines the effects of an output cap and an upper reflecting barrier on demand. The output cap arises from the finite output capacity of the firm, leading to a portion of the potential demand remaining unsatisfied. Simultaneously, the reflecting barrier captures the realistic scenario in which part of the unsatisfied demand opts for alternative consumption choices. This model is particularly relevant in the context of infrastructure projects, such as airports. The main findings reveal that an upper reflecting barrier significantly affects leverage decisions, while its impact on investment timing decisions is less pronounced. The effects become more prominent under higher uncertainty. When the option to expand exists, eliminating both the cap and the barrier, initial investment accelerates and leverage ratios and credit spreads decrease.

Keywords: Caps; Capital structure; Investment decisions; Real options
JEL codes: G31; G32; D81.

*Paulo J. Pereira and Artur Rodrigues acknowledge that this research has been financed by Portuguese public funds through FCT - Fundação para a Ciência e a Tecnologia, I.P., in the framework of the projects UIDB/04105/2020 and UIDB/03182/2020, respectively.

1 Introduction

Infrastructure projects play a central role in modern economies, promoting growth and enhancing development. An important aspect when undertaking such projects is the relationship between the supply constraints inherent in the infrastructure and the potential demand. In this paper, we study the effects of an output cap, which captures a supply constraint, and an upper reflecting barrier on demand, affecting shadow demand.

The concept of an output cap arises from the typical finite capacity of firms involved in infrastructure development. It introduces a fundamental constraint, since part of the potential demand remains unsatisfied due to the firm's inability to meet it fully. Simultaneously, the presence of an upper reflecting barrier introduces a new layer of complexity by capturing the case in which some of this unsatisfied demand, when it exceeds the barrier level, opts for alternative consumption choices. This phenomenon is particularly relevant in the context of infrastructure projects, such as airports, where demand may surpass the available capacity, requiring a detailed analysis of its impact.

The subject of caps with unconstrained (shadow) demand has been studied in the literature. Dixit (1991) examines the effect of price ceilings on irreversible investment decisions under uncertainty. He demonstrates that in the absence of external restrictions, investment alone can prevent prices from exceeding a natural ceiling. When a lower price ceiling is enforced, investment occurs only if a substantially higher "shadow" price is observed. If the set ceiling decreases to match the long-run average cost, the shadow price goes to infinity, leading to a complete cessation of investment.

The effects of price regulation (via price caps) in firms with market power is studied by Dobbs (2004). Under uncertainty, investment is delayed, leading to higher prices over time, compared to a similar competitive industry. Under certainty, applying an intertemporal price cap can produce outcomes analogous to those of a competitive market. The authors show this conclusion does not stand under uncertainty.

In Evans and Guthrie (2012) it is shown that regulated firms, both in terms of price and quantity, tend to make investments in smaller and more frequent increments compared to social planners. Furthermore, these investment distortions become more pronounced with larger economies of scale.

More recently, Adkins et al. (2019) build on the existing literature on collar-style agreements (involving floors and ceilings) by offering a solution for the case of finite-lived collars. The authors demonstrate that finite-lived collars significantly influence optimal behavior, specifically by inducing earlier investment timing, compared to perpetual collars.

Sarkar (2016) and Rodrigues (2023) add financing decisions to this branch of literature. The former paper investigates the effect of price caps on consumer welfare by considering indirect effects of regulation on investment and leverage decisions. Paradoxically, it shows that a more consumer-friendly regulator might result in a lower level of consumer welfare.

In fact, a price cap can negatively impact investment, leading to a reduction in long-term consumer welfare through the restriction of supply. The latter paper explores how caps and floors affect investment timing, leverage decisions, firm value, and credit spreads. It is shown that floors below a critical level moderately affect leverage, while exceeding this threshold promotes reduced-risk debt issuance and faster investments. Lower caps, however, deter investment, increase leverage, and reduce credit spreads. In a collar regime, the interaction between caps and floors leads to varied effects on debt issuance. Overall, uncertainty generally discourages investment and has complex impacts on leverage and credit spreads.

All these papers model the unconstrained demand as a shadow stochastic process with a cap limiting supply and setting the effective demand. An alternative approach to model the constrained effective demand is adding an upper reflecting barrier to the stochastic process. An early study that includes reflecting barriers appears in Dixit and Pindyck (1994). Here, prices range between an upper and a lower reflecting barrier due to the entry or exit of firms in the industry.

In a recent development, Nishihara and Shibata (2023) analyzes the optimal capital structure for a firm with uncertain earnings, incorporating a lower reflecting barrier (e.g, a price floor or a threshold). Contrary to standard unrestricted models, under this setup the firm is able to issue risk-free debt up to a limit set by the lower barrier. Higher lower barriers enable more risk-free debt, encouraging a riskless capital over a risky capital structure. At moderate barrier levels, the firm might opt for a risk-free capital structure with lower leverage than if there were no barrier, which might explain the observed debt conservatism.

Our paper differs from the previous literature in several ways. First, we consider the joint effect of output caps, upper reflecting barriers and financing decisions. This setting offers useful insights for decision-making processes, namely regarding the investment timing of infrastructure projects. It also offers an understanding of how supply and demand constraints influence the policy for financing the project, focusing on aspects such as leverage, default dynamics, and credit spreads.

Furthermore, we extend the model to consider the possibility of expanding the scale of the project. This expansion enables the firm to eliminate or at least mitigate the effect of both the output cap and the upper reflecting barrier. We will analyze in detail the impact of this expansion option on the dynamics of the initial investment and the financing policy.

The main results indicate that an upper reflecting barrier significantly affects leverage decisions, while its impact on investment timing decisions is less pronounced. The effects of the barrier become more significant under higher uncertainty. When the option to expand is available, initial investment accelerates, and leverage ratios and credit spreads decrease.

The paper unfolds as follows: after this introduction, Section 2 presents the models for different settings: caps only, caps and barriers both for levered and unlevered firms, and the model that includes the option to expand is also derived. Section 4 analyzes the main outcomes of the models using a numerical example. Finally, Section 5 concludes.

2 Investment and leverage decisions subject to constrained demand

In this section, we study the effects of constrained demand on investment policy. In particular, we analyze how restrictions on the firm's output impact the timing and leverage of the project. Two types of restrictions are considered. First, when the constraint is in the form of a cap, limiting the output offered by the firm; and then, along with the cap, a reflecting barrier also restricts the potential demand of the firm. In other words, the cap limits the quantities that can be offered by the firm, whereas the barrier restricts the dynamics of the shadow demand. Let us start with the simple case where only a cap is considered.

2.1 Output constrained by a cap

We consider first the case of an unlevered firm and then we discuss the financing policy in the presence of a cap.

Unlevered firm

The firm's profit ($\pi(t)$) depends on both the instantaneous quantity of demand (Q) and on the unitary EBIT margin (m). The quantity Q depends on the dynamics of demand and is assumed to behave randomly according to a geometric Brownian motion (gBm):

$$dQ = \alpha Q dt + \sigma Q dW \tag{1}$$

where $\alpha < r$ represents the instantaneous risk-neutral drift, r represents the risk-free interest rate, σ represents the instantaneous volatility, and dW denotes the standard Wiener increment.

In the case of an unconstrained demand, the after-tax profit flow is:

$$\pi(t) = mQ(1 - \tau) \tag{2}$$

where τ is the corporate tax rate.

Consider now the case where the firm has a maximum output capacity (i.e., the firm has a limiting output level corresponding to a cap C). In such a context, the profit is subject to the maximum mC, as the firm is not be able to satisfy any demand exceeding C. In fact, if demand is larger than the cap, $Q > C$, the output will correspond to C, and the firm will not able to satisfy a demand of $Q - C$. For instance, consider the case of an airport where the number of passengers is limited to the capacity of the infrastructure. If the number of (potential) passengers exceeds the airport's capacity, some of the demand will end up unsatisfied, as some passengers are unable to find available flights.

Under an output cap (C), the profit flow for the firm has an upper limit, as follows:

$$\pi^{cap}(t) = m \min\{Q, C\}(1-\tau), \tag{3}$$

Given that Q evolves randomly, if $Q < C$ (or $Q > C$), the firm's value function needs to account for the possibility of Q becoming larger (or smaller) than C in a future moment. The contingent claims approach is used to derive the value function of the active project for the full domain of Q, as well as the value and the decision rule for the project in the idle stage.

Proposition 1. *Under a cap on the firm's output, the value of the active unlevered firm is:*

$$V_u^{cap} = \begin{cases} G_{11}^{cap} Q^{\beta_1} + \dfrac{mQ}{r-\alpha}(1-\tau) & \text{for } Q < C \\ G_{22}^{cap} Q^{\beta_2} + \dfrac{mC}{r}(1-\tau) & \text{for } Q \geqslant C \end{cases} \tag{4}$$

where

$$G_{11}^{cap} = \frac{mC^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{r-\alpha} - \frac{\beta_2}{r}\right)(1-\tau) \tag{5}$$

$$G_{22}^{cap} = \frac{mC^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r}\right)(1-\tau) \tag{6}$$

While in the idle stage, the firm optimally undertakes the project when the investment threshold is achieved. Notice that, depending on the investment cost K, the threshold can either be below or above the cap (C). The amount of K that separates the two regions is as follows:

$$K_u^{cap} = \frac{\beta_1 - 1}{\beta_1}\frac{mC}{r-\alpha}(1-\tau) \tag{7}$$

Accordingly, if $K < K_u^{cap}$ the investment threshold is:

$$Q_u^{cap} = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{1-\tau}\frac{K}{m} < C \tag{8}$$

and if $K > K_u^{cap}$ the threshold becomes:

$$Q_u^{cap} = \left(\frac{\beta_1}{(\beta_1 - \beta_2)G_{22}^{cap}}\left(K - \frac{mC}{r}(1-\tau)\right)\right)^{1/\beta_2} > C \tag{9}$$

and, naturally, when $K = K_u^{cap}$ then $Q_u^{cap} = C$.

Finally, the value of the firm in the idle stage, i.e., before the investment threshold is reached ($Q < Q_u^{cap}$), is:

$$F_u^{cap}(Q) = (V_u^{cap}(Q_u^{cap}) - K)\left(\frac{Q}{Q_u^{cap}}\right)^{\beta_1} \tag{10}$$

Levered firm

Consider now the case of a levered firm. In this setting, the flexibility to default on debt needs to be considered. Following Leland (1994), shareholders are assumed to have 'deep pockets'. Profits are distributed as dividends, and losses are financed by issuing new equity (with no emission costs). When profits drop to a sufficiently low level, it is optimal, from the shareholders' perspective, to default by not paying the perpetual coupon. In the case of default, debtholders receive a fraction of the unlevered firm and incur a proportional default cost ϕ. Under this setting, the after-tax profit flow for shareholders is as follows:

$$\pi_l^{cap}(t) = (m \min\{Q, C\} - c)(1-\tau), \tag{11}$$

where c is a perpetual coupon paid to debtholders. Using standard contingent claims arguments, the investment opportunity is characterized as follows:

Proposition 2. *Under the output cap, the value of the active levered firm for the equity-holders is:*

$$E^{cap}(Q,c) = \begin{cases} 0 & Q < Q_d^{cap}(c) \\ E_{11}^{cap} Q^{\beta_1} + E_{12}^{cap}(c) Q^{\beta_2} + \left(\frac{mQ}{r-\alpha} - \frac{c}{r}\right)(1-\tau) & Q_d^{cap}(c) \leqslant Q < C \\ E_{22}^{cap}(c) Q^{\beta_2} + \left(\frac{mC}{r} - \frac{c}{r}\right)(1-\tau) & Q \geqslant C \end{cases} \tag{12}$$

where

$$E_{11}^{cap} = G_{11}^{cap} \tag{13}$$

$$E_{12}^{cap}(c) = \frac{\left(Q_d^{cap}(c)\right)^{-\beta_2}}{\beta_2 - \beta_1}\left((\beta_1 - 1)\frac{mQ_d^{cap}(c)}{r-\alpha} - \beta_1 \frac{c}{r}\right)(1-\tau) \tag{14}$$

$$E_{22}^{cap}(c) = G_{22}^{cap} + E_{12}^{cap}(c) \tag{15}$$

The value of the debt, considering that debtholders receive $(1-\phi)V_u^{cap}$ upon default, is:

$$D^{cap}(Q,c) = \frac{c}{r} + D_2^{cap}(c) Q^{\beta_2} \tag{16}$$

where

$$D_2^{cap}(c) = \left((1-\phi)\left(G_{11}^{cap}\left(Q_d^{cap}(c)\right)^{\beta_1} + \frac{mQ_d^{cap}(c)(1-\tau)}{r-\alpha}\right) - \frac{c}{r}\right)\left(\frac{1}{Q_d^{cap}(c)}\right)^{\beta_2} \quad (17)$$

and the optimal threshold to default, $Q_d^{cap}(c)$*, solves the following equation:*

$$(\beta_1-\beta_2)E_{11}^{cap}(Q_d^{cap}(c))^{\beta_1} + \left((1-\beta_2)\frac{mQ_d^{cap}(c)}{r-\alpha} + \beta_2\frac{c}{r}\right)(1-\tau) = 0 \quad (18)$$

Shareholders invest at the optimal threshold Q_l^{cap} *and choose the the optimal coupon* c^{cap} *by maximizing the value of the idle firm:*

$$\{Q_l^{cap}, c^{cap}\} = \underset{\{Q_l,c\}}{\operatorname{argmax}}\left(E^{cap}(Q_l,c) + D^{cap}(Q_l,c) - K\right)\left(\frac{Q}{Q_l}\right)^{\beta_1} \quad (19)$$

2.2 Output constrained by a cap and demand constrained by an upper barrier

In the previous section, we did not consider any constraints affecting the potential demand of the firm. In such a scenario, all unsatisfied consumers are considered as potential future customers. For example, consider again the case of an airport: when demand exceeds capacity, some passengers are unable to travel. However, all these passengers would travel in the event of a sudden increase in capacity. It is reasonable to assume, however, that this may not always be the case. In fact, when some of the unsatisfied demand chooses alternative consumption options, the shadow demand is constrained by an upper reflecting barrier (B, where $B > C$), representing its maximum limit.

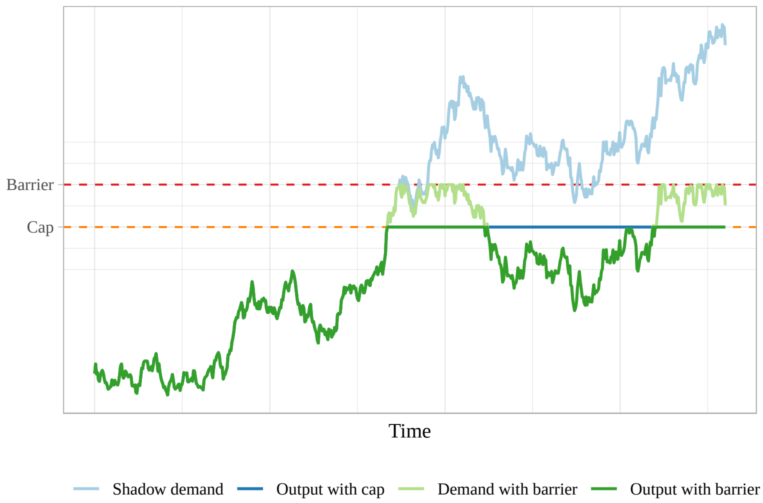


Figure 1: Effect of a cap and a barrier on the effective output.

The presence of a reflecting barrier on the demand has a significant impact on the project's value. This is because the barrier diminishes the probability of the firm earning its maximum possible profit in the future. In simple terms, the duration over which the firm can expect to receive its maximum profit (when the output is the cap) is shorter when there is a reflecting barrier. This effect is illustrated in Figure 1. As we can see, in the absence of a barrier, the output attains its maximum (cap) after demand hts the cap for the first time, while the introduction of an upper barrier only allows the firm to achieve the maximum output for shorter periods.

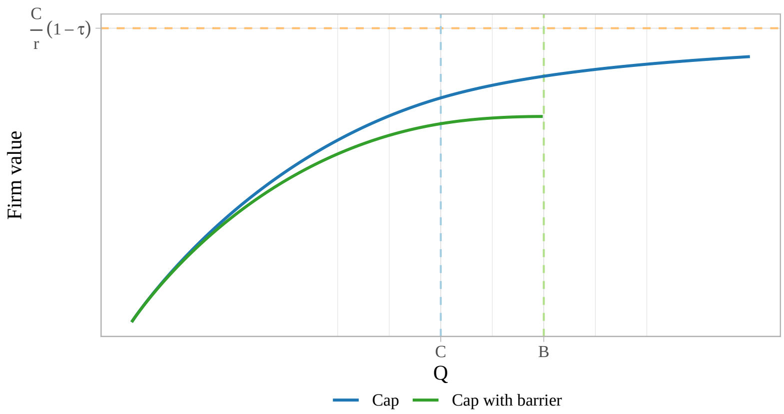


Figure 2: Effect of a cap and a barrier on the unlevered firm value.

The effect of the cap and barrier on the firm's value is illustrated in Figure 2, that depicts the case of an unlevered firm. The figure shows how the value of firm changes as the shadow unrestricted demand changes (Q). The cap on demand limits and reduces the value of the firm. The addition of an upper reflecting barrier, that restricts demand, reduces even further the firm value.

Unlevered firm

As in the previous section, let us start with the case where the firm is fully financed with equity.

Proposition 3. *Under the joint effect of a cap on output and a barrier on the demand, the value of the active unlevered firm is:*

$$V_u^{bar} = \begin{cases} G_{11}^{bar} Q^{\beta_1} + \dfrac{mQ}{r-\alpha}(1-\tau) & Q < C \\ G_{21}^{bar} Q^{\beta_1} + G_{22}^{bar} Q^{\beta_2} + \dfrac{mC}{r}(1-\tau) & C \leqslant Q \leqslant B \end{cases} \tag{20}$$

where

$$G_{11}^{bar} = G_{21}^{bar} + G_{11}^{cap} \tag{21}$$

$$G_{21}^{bar} = \frac{\beta_2 B^{\beta_2}}{\beta_1 B^{\beta_1}} G_{22}^{cap} \tag{22}$$

$$G_{22}^{bar} = G_{22}^{cap} \tag{23}$$

and G_{11}^{cap} *and* G_{22}^{cap}, *are given by Equations* (5) *and* (6), *respectively.*

While in the idle stage, the firm optimally undertakes the project when the investment threshold is achieved. As before, depending on the investment cost K, *the threshold value can either be below or above the cap* (C). *The amount of* K *that separates the two regions is the same as in Equation* (7), *i.e.:*

$$K_u^{bar} = K_u^{cap} \tag{24}$$

Accordingly, if $K < K_u^{bar}$ *the investment threshold is:*

$$Q_u^{bar} = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{1-\tau} \frac{K}{m} = Q_u^{cap} < C \tag{25}$$

and if $K >_u^{bar}$ *the threshold becomes:*

$$Q_u^{bar} = \min\left[\left(\frac{\beta_1}{(\beta_1 - \beta_2) G_{22}^{bar}} \left(K - \frac{mC}{r}(1-\tau)\right)\right)^{1/\beta_2} = Q_u^{cap}, B\right] \tag{26}$$

and, naturally, when $K = K_u^{bar}$ *then* $Q_u^{bar} = C$.

The value of the firm while in the idle stage, i.e., before reaching the investment threshold ($Q < Q_u^{bar}$), *is:*

$$F_u^{bar}(Q) = \left(V_u^{bar}(Q_u^{bar}) - K\right)\left(\frac{Q}{Q_u^{bar}}\right)^{\beta_1} \tag{27}$$

For the unlevered firm, an upper reflecting barrier, added to an output cap, naturally reduces its value but has no effect on investment timing decisions, which are only determined by the output cap, except when the threshold is the barrier. That occurs when the barrier is sufficiently close to the cap or when the investment cost K is large enough, i.e. when

$$K > \left(1 - \frac{\beta_2}{\beta_1}\right) G_{22}^{bar} B^{\beta_2} + \frac{mC}{r}(1-\tau) > K_u^{cap} \tag{28}$$

Levered firm

For the case of a levered firm, the solution is described as follows:

Proposition 4. *Under both the output cap and the demand barrier, the value of the active levered firm for the equityholders is:*

$$E^{bar}(Q,c) = \begin{cases} 0 & Q < Q_d^{bar}(c) \\ E_{11}^{bar}(c)Q^{\beta_1} + E_{12}^{bar}(c)Q^{\beta_2} + \left(\dfrac{mQ}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) & Q_d^{bar}(c) \leqslant Q < C \\ E_{21}^{bar}(c)Q^{\beta_2} + E_{22}^{bar}(c)Q^{\beta_2} + \left(\dfrac{mC}{r} - \dfrac{c}{r}\right)(1-\tau) & C \leqslant Q \leqslant B \end{cases} \tag{29}$$

where

$$E_{11}^{bar}(c) = G_{11}^{bar} - \frac{\beta_2 B^{\beta_2}}{\beta_1 B^{\beta_1}} E_{12}^{bar}(c) \tag{30}$$

$$E_{12}^{bar}(c) = \frac{\left(Q_d^{bar}(c)\right)^{-\beta_2}}{\beta_2 - \beta_1} \left((\beta_1 - 1)\frac{mQ_d^{bar}(c)}{r-\alpha} - \beta_1 \frac{c}{r}\right)(1-\tau) \tag{31}$$

$$E_{21}^{bar}(c) = G_{21}^{bar} - \frac{\beta_2 B^{\beta_2}}{\beta_1 B^{\beta_1}} E_{12}^{bar}(c) \tag{32}$$

$$E_{22}^{bar}(c) = G_{22}^{bar} + E_{12}^{bar}(c) \tag{33}$$

The value of the debt, considering that debtholders receive $(1-\phi)V_u^{bar}$ upon default, is:

$$D^{bar}(Q,c) = \frac{c}{r} + D_1^{bar}(c)Q^{\beta_1} + D_2^{bar}(c)Q^{\beta_2} \tag{34}$$

where

$$D_1^{bar}(c) = -\beta_2 B^{\beta_2} z(c) \left(\frac{1}{Q_d^{bar}(c)}\right)^{\beta_1} \tag{35}$$

$$D_2^{bar}(c) = \beta_1 B^{\beta_1} z(c) \left(\frac{1}{Q_d^{bar}(c)}\right)^{\beta_2} \tag{36}$$

and

$$z(c) = \frac{1}{\beta_1 B^{\beta_1}(Q_d^{bar}(c))^{\beta_2} - \beta_2 B^{\beta_2}(Q_d^{bar}(c))^{\beta_1}} \times \left((1-\phi)\left(G_{11}^{bar}(Q_d^{bar}(c))^{\beta_1} + \frac{Q_d^{bar}(c)(1-\tau)}{r-\alpha}\right) - \frac{c}{r}\right) \tag{37}$$

and the optimal threshold to default, $Q_d^{bar}(c)$, solves the following equation:

$$(\beta_1 - \beta_2)E_{11}^{bar}(c)(Q_d^{bar}(c))^{\beta_1} + \left((1-\beta_2)\frac{mQ_d^{bar}(c)}{r-\alpha} + \beta_2 \frac{c}{r}\right)(1-\tau) = 0 \tag{38}$$

Shareholders optimally invest at the threshold Q_l^{bar} and choose the the optimal coupon c^{bar}. Both Q_l^{bar} and c^{bar} are those that maximize the value of the idle firm:

$$\{Q_l^{bar}, c^{bar}\} = \underset{\{Q_l,c\}}{\operatorname{argmax}} \left(E^{bar}(Q_l, c) + D^{bar}(Q_l, c) - K\right)\left(\frac{Q}{Q_l}\right)^{\beta_1} \tag{39}$$

Contrary to the case of the levered firm, the barrier has not only an effect of the firm's securities values but it always induces different investment and leverage decisions.

3 Option to expand and remove the restrictions

In this section, we extend the model by considering the investment to expand capacity. For the sake of simplicity, we assume that the expansion it to an unlimited capacity, thereby removing the cap on the output. For an expansion cost of K_e, the firm is able to eliminate the upper bound in the output. As before, we separate the analysis, considering the cases of unlevered and levered firms. In Figure 3 we can see the effect of the reflecting barrier at the moment of expansion. If no barrier restricts the shadow demand, at the time of expansion, the quantity resumes at the level of the shadow demand. If the barrier is present, the quantity will start at a lower level, due to the reflecting effect of the barrier.

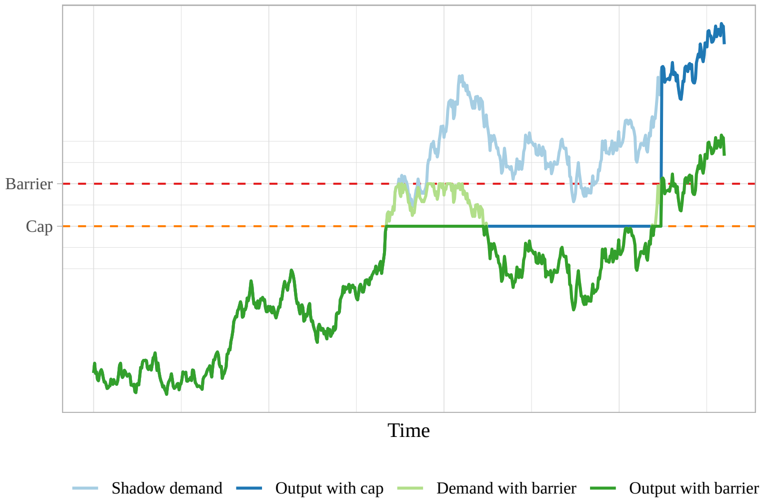


Figure 3: Effect of a cap and a barrier on the effective output.

With an unlimited capacity after expansion and an unconstrained demand, the profit flows of the unlevered and levered firm become $mQ(1-\tau)$ and $(mQ-c)(1-\tau)$, respectively.

3.1 Output constrained by a cap

Unlevered firm

It can be shown that the expansion threshold never lies below the cap C, i.e. it is never optimal for the firm to expand, eliminating the cap, for a demand lower than C. The investment policy is as follows:

Proposition 5. *The payoff of expansion at the threshold is:*

$$\Pi_{ue}^{cap} = \frac{mQ_{ue}^{cap}}{r-\alpha}(1-\tau) - G_{22}^{cap}(Q_{ue}^{cap})^{\beta_2} - \frac{mC}{r}(1-\tau) - K_e \tag{40}$$

where the expansion threshold Q_e^{cap} solves the equation:

$$-(\beta_1-\beta_2)G_{22}^{cap}(Q_{ue}^{cap})^{\beta_2} + (\beta_1-1)\frac{mQ_{ue}^{cap}}{r-\alpha}(1-\tau) - \beta_1\left(K_e + \frac{mC}{r}(1-\tau)\right) = 0 \tag{41}$$

Moving back to the stage before expansion, the value of the active unlevered firm is:

$$V_u^{e-cap} = V_u^{cap} + \Pi_{ue}^{cap}\left(\frac{Q}{Q_{ue}^{cap}}\right)^{\beta_1} \tag{42}$$

where the last term is the value of the option to expand, which increases the value fo the unlevered firm in this first stage. However, the investment decision at the idle stage has the same threshold as that of the project without expansion ($Q_u^{e-cap} = Q_u^{cap}$), i.e. as defined by Equations (7)–(9).

In summary, for an unlevered firm with the output capped by its maximum capacity, the possibility of capacity expansion increases the project value, but does not change the initial investment timing.

Levered firm

We assume that the expansion creates another opportunity, after the initial investment, for the firm to optimize its capital structure. It is assumed that the debt is recalled at the market value and new debt is issued.

Let us begin by presenting the value of the firm's securities after expansion, when the firm is not under any output constraint:

Proposition 6. *The value for the equityholders is:*

$$E(Q,c) = \begin{cases} 0 & Q < Q_d(c) \\ E_2(c)Q^{\beta_2} + \left(\frac{mQ}{r-\alpha} - \frac{c}{r}\right)(1-\tau) & Q \geqslant Q_d(c) \end{cases} \tag{43}$$

where

$$E_2(c) = \left(\frac{c}{r} - \frac{mQ_d}{r-\alpha}\right)(1-\tau)\left(\frac{1}{Q_d}\right)^{\beta_2} \tag{44}$$

and Q_d is the threshold for default:

$$Q_d(c) = \frac{\beta_2}{\beta_2 - 1}\left(\frac{r-\alpha}{m}\right)\frac{c}{r} \tag{45}$$

The value for the debtholders is:

$$D(Q,c) = \frac{c}{r} + D_2(c)Q^{\beta_2} \tag{46}$$

where

$$D_2(c) = \left((1-\phi)\left(\frac{mQ_d(1-\tau)}{r-\alpha}\right) - \frac{c}{r}\right)\left(\frac{1}{Q_d(c)}\right)^{\beta_2} \tag{47}$$

Equityholders choose the the optimal coupon c_e^{cap} by maximizing the value of the firm at the expansion threshold Q_{le}^{cap}:

$$c_e^{cap} = \frac{\beta_2 - 1}{\beta_2}\left(\frac{r}{r-\alpha}\right)\frac{mQ_{le}^{cap}}{h} \tag{48}$$

where

$$h = \left(1 - \beta_2\left(1 - \phi + \frac{\phi}{\tau}\right)\right)^{1/\beta_2} < 1 \tag{49}$$

The following proposition states the investment and financing policies of the firm under a cap in output and an option to expand:

Proposition 7. *Depending on the cost of expansion, the threshold for expansion (Q_{le}^{cap}),may be either below or above C.*

i. When expansion occurs above the cap ($Q_{le}^{cap} > C$), the value for the equityholders is:

$$E^{e-cap}(Q,c) = \begin{cases} 0 & Q < Q_d^{e-cap}(c) \\ E_{11}^{e-cap}(c)Q^{\beta_1} + E_{12}^{e-cap}(c)Q^{\beta_2} & \\ \quad + \left(\frac{mQ}{r-\alpha} - \frac{c}{r}\right)(1-\tau) & Q_d^{e-cap}(c) \leqslant Q < C \\ E_{21}^{e-cap}(c)Q^{\beta_2} + E_{22}^{e-cap}(c)Q^{\beta_2} & \\ \quad + \left(\frac{mC}{r} - \frac{c}{r}\right)(1-\tau) & C \leqslant Q < Q_{le}^{cap} \\ E(Q, c_e^{cap})Q^{\beta_2} + D(Q, c_e^{cap}) & \\ \quad - D^{e-cap}(Q,c) - K_e & Q \geqslant Q_{le}^{cap} \end{cases}$$

where

$$E_{11}^{e-cap}(c) = G_{11}^{cap} + E_{21}^{e-cap}(c) \tag{50}$$

$$E_{12}^{e-cap}(c) = -\left(E_{11}^{e-cap}(c)(Q_d^{e-cap})^{\beta_1} + \left(\frac{mQ_d^{e-cap}}{r-\alpha} - \frac{c}{r}\right)(1-\tau)\right)(Q_d^{e-cap})^{-\beta_2} \tag{51}$$

$$\begin{aligned} E_{21}^{e-cap}(c) = \Bigg(&\left(E_2(c_e^{cap}) + D_2(c_e^{cap}) - E_{22}^{e-cap}(c) - D_2^{e-cap}(c)\right)(Q_{le}^{cap})^{\beta_2} \\ &+ \left(\frac{Q_{le}^{cap}}{r-\alpha} - \frac{C}{r}\right)m(1-\tau) + \frac{c_e^{cap}-c}{r}\tau - K_e\Bigg)(Q_{le}^{cap})^{-\beta_1} \end{aligned} \tag{52}$$

$$E_{22}^{e-cap}(c) = G_{22}^{cap} + E_{12}^{e-cap}(c) \tag{53}$$

where the expansion threshold Q_{le}^{cap} solves the equation:

$$\begin{aligned} &(\beta_1-\beta_2)\left(E_2(c_e^{cap}) + D_2(c_e^{cap}) - E_{22}^{e-cap}(c) - D_2^{e-cap}(c)\right)(Q_{le}^{cap})^{\beta_2} \\ &\quad + (\beta_1-1)\frac{mQ_{le}^{cap}}{r-\alpha}(1-\tau) - \beta_1\left(K_e + \frac{mC}{r}(1-\tau) - \frac{c_e^{cap}-c}{r}\tau\right) = 0 \end{aligned} \tag{54}$$

and the default threshold is the solution to the following equation:

$$(\beta_1-\beta_2)E_{11}^{e-cap}(c)(Q_d^{e-cap}(c))^{\beta_1} + \left((1-\beta_2)\frac{mQ_d^{e-cap}(c)}{r-\alpha} + \beta_2\frac{c}{r}\right)(1-\tau) = 0 \tag{55}$$

ii. When expansion occurs below the cap ($Q_{le}^{cap} < C$), the value for the equityholders is:

$$E^{e-cap}(Q,c) = \begin{cases} 0 & Q < Q_d^{e-cap}(c) \\ E_{11}^{e-cap}(c)Q^{\beta_1} + E_{12}^{e-cap}(c)Q^{\beta_2} \\ \qquad + \left(\frac{mQ}{r-\alpha} - \frac{c}{r}\right)(1-\tau) & Q_d^{e-cap}(c) \leqslant Q < Q_{le}^{cap} \\ E(Q,c_e^{cap})Q^{\beta_2} + D(Q,c_e^{cap}) \\ \qquad - D^{e-cap}(Q,c) - K_e & Q \geqslant Q_{le}^{cap} \end{cases}$$

where

$$\begin{aligned} E_{11}^{e-cap}(c) = \Bigg(&\left(E_2(c_e^{cap}) + D_2(c_e^{cap}) - E_{12}^{e-cap}(c) - D_2^{e-cap}(c)\right)(Q_{le}^{cap})^{\beta_2} \\ &+ \frac{c_e^{cap}-c}{r}\tau - K_e\Bigg)(Q_{le}^{cap})^{-\beta_1} \end{aligned} \tag{56}$$

$$E_{12}^{e-cap}(c) = -\left(E_{11}^{e-cap}(c)(Q_d^{e-cap})^{\beta_1} + \left(\frac{mQ_d^{e-cap}}{r-\alpha} - \frac{c}{r}\right)(1-\tau)\right)(Q_d^{e-cap}(c))^{-\beta_2} \tag{57}$$

where the expansion threshold Q_{le}^{cap} is:

$$Q_{le}^{cap} = \left(\frac{\beta_1}{(\beta_1 - \beta_2)\left(E_2(c_e^{cap}) + D_2(c_e^{cap}) - E_{12}^{e-cap}(c) - D_2^{e-cap}(c)\right)} \left(K_e - \frac{c_e^{cap} - c}{r}\tau \right) \right)^{1/\beta_2} \tag{58}$$

and the default threshold is the solution to the following equation:

$$(\beta_1 - \beta_2)E_{11}^{e-cap}(c)(Q_d^{e-cap}(c))^{\beta_1} + \left((1-\beta_2)\frac{mQ_d^{e-cap}(c)}{r-\alpha} + \beta_2\frac{c}{r} \right)(1-\tau) = 0 \tag{59}$$

The value of the debt, considering that debtholders receive $(1-\phi)V_u^{e-cap}$ upon default, is:

$$D^{e-cap}(Q,c) = \frac{c}{r} + D_1^{e-cap}(c)Q^{\beta_1} + D_2^{e-cap}(c)Q^{\beta_2} \tag{60}$$

where

$$D_1^{e-cap}(c) = y(c)\left((Q_{le}^{cap})^{\beta_2}w(c) + (Q_d^{e-cap}(c))^{\beta_2}D_2(c_e^{cap})(Q_{le}^{cap})^{\beta_2}\right) \tag{61}$$

$$D_2^{e-cap}(c) = -y(c)\left((Q_{le}^{cap})^{\beta_1}w(c) + (Q_d^{e-cap}(c))^{\beta_1}D_2(c_e^{cap})(Q_{le}^{cap})^{\beta_2}\right) \tag{62}$$

and

$$y(c) = \frac{1}{(Q_d^{e-cap}(c))^{\beta_2}(Q_{le}^{cap})^{\beta_1} - (Q_d^{e-cap}(c))^{\beta_1}(Q_{le}^{cap})^{\beta_2}} \tag{63}$$

$$w(c) = -\left((1-\phi)\left(\left(G_{11}^{cap} + \Pi_{ue}^{cap}\left(\frac{1}{Q_{ue}^{cap}}\right)^{\beta_1} \right)(Q_d^{e-cap}(c))^{\beta_1} + \frac{Q_d^{e-cap}(c)(1-\tau)}{r-\alpha} \right) - \frac{c}{r} \right) \tag{64}$$

Shareholders optimally invest at the threshold Q_{le}^{cap} and choose the the optimal coupon c^{e-cap}, that maximize the value of the idle firm:

$$\{Q_{le}^{cap}, c^{e-cap}\} = \underset{\{Q_l,c\}}{\operatorname{argmax}} \left(E^{e-cap}(Q_l,c) + D^{e-cap}(Q_l,c) - K\right)\left(\frac{Q}{Q_l}\right)^{\beta_1} \tag{65}$$

3.2 Output constrained by a cap and demand constrained by an upper barrier

This section presents the value of unlevered and levered firms when, additionally to an output cap, demand is constrained by an upper reflecting barrier.

Unlevered firm

As before, expansion of the unlevered firm can only occur above the cap. The investment policy is described in the following proposition:

Proposition 8. *The payoff of expansion at the threshold is:*

$$\Pi_{ue}^{bar} = \frac{mQ_{ue}^{bar}}{r-\alpha}(1-\tau) - G_{21}^{bar}(Q_{ue}^{bar})^{\beta_1} - G_{22}^{bar}(Q_{ue}^{bar})^{\beta_2} - \frac{mC}{r}(1-\tau) - K_e \tag{66}$$

where the expansion threshold Q_e^{bar} solves the equation:

$$-(\beta_1-\beta_2)G_{22}^{bar}(Q_{ue}^{bar})^{\beta_2} + (\beta_1-1)\frac{mQ_{ue}^{bar}}{r-\alpha}(1-\tau) - \beta_1\left(K_e + \frac{mC}{r}(1-\tau)\right) = 0 \tag{67}$$

In the idle stage before expansion, the value of the active unlevered firm is:

$$V_u^{e-bar} = V_u^{bar} + \Pi_{ue}^{bar}\left(\frac{Q}{Q_{ue}^{bar}}\right)^{\beta_1} \tag{68}$$

where the last term is the value of the option to expand, which increases the value fo the unlevered firm in this first stage. As for the cap case, the investment thresholds of the first stage are not affect by the option to expand ($Q_u^{e-bar} = Q_u^{bar}$), i.e. they are defined by Equations (25) *and* (26)*.*

Similarly to the cap case, the unlevered firm does not change the initial investment timing when an option to expand is available.

Levered firm

The following proposition states the investment and financing policies of the firm under a cap in output and demand constrained by a barrier when expansion is possible::

Proposition 9. *Depending on the cost of expansion, the threshold for expansion (Q_{le}^{bar}),may be either below or above C.*

i. When expansion occurs above the cap ($Q_{le}^{bar} > C$), the value for the equityholders is:

$$E^{e-bar}(Q,c) = \begin{cases} 0 & Q < Q_d^{e-bar}(c) \\ E_{11}^{e-bar}(c)Q^{\beta_1} + E_{12}^{e-bar}(c)Q^{\beta_2} \\ \quad + \left(\dfrac{mQ}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) & Q_d^{e-bar}(c) \leqslant Q < C \\ E_{21}^{e-bar}(c)Q^{\beta_2} + E_{22}^{e-bar}(c)Q^{\beta_2} \\ \quad + \left(\dfrac{mC}{r} - \dfrac{c}{r}\right)(1-\tau) & C \leqslant Q < Q_{le}^{bar} \\ E(Q, c_e^{bar})Q^{\beta_2} + D(Q, c_e^{bar}) \\ \quad - D^{e-bar}(Q,c) - K_e & Q \geqslant Q_{le}^{bar} \end{cases}$$

where

$$E_{11}^{e-bar}(c) = G_{11}^{cap} + E_{21}^{e-bar}(c) \tag{69}$$

$$E_{12}^{e-bar}(c) = -\left(E_{11}^{e-bar}(c)(Q_d^{e-bar})^{\beta_1} + \left(\frac{mQ_d^{e-bar}}{r-\alpha} - \frac{c}{r}\right)(1-\tau)\right)(Q_d^{e-bar})^{-\beta_2} \tag{70}$$

$$\begin{aligned} E_{21}^{e-bar}(c) = \Bigg(&\left(E_2(c_e^{bar}) + D_2(c_e^{bar}) - E_{22}^{e-bar}(c) - D_2^{e-bar}(c)\right)(Q_{le}^{bar})^{\beta_2} \\ &+ \left(\frac{Q_{le}^{bar}}{r-\alpha} - \frac{C}{r}\right)m(1-\tau) + \frac{c_e^{bar} - c}{r}\tau - K_e\Bigg)(Q_{le}^{bar})^{-\beta_1} \end{aligned} \tag{71}$$

$$E_{22}^{e-bar}(c) = G_{22}^{cap} + E_{12}^{e-bar}(c) \tag{72}$$

where the expansion threshold Q_{le}^{bar} solves the equation:

$$\begin{aligned} (\beta_1 - \beta_2)&\left(E_2(c_e^{bar}) + D_2(c_e^{bar}) - E_{22}^{e-bar}(c) - D_2^{e-bar}(c))\right)(Q_{le}^{bar})^{\beta_2} \\ &+ (\beta_1 - 1)\frac{mQ_{le}^{bar}}{r-\alpha}(1-\tau) - \beta_1\left(K_e + \frac{mC}{r}(1-\tau) - \frac{c_e^{bar} - c}{r}\tau\right) = 0 \end{aligned} \tag{73}$$

and the default threshold is the solution to the following equation:

$$(\beta_1 - \beta_2)E_{11}^{e-bar}(c)(Q_d^{e-bar}(c))^{\beta_1} + \left((1-\beta_2)\frac{mQ_d^{e-bar}(c)}{r-\alpha} + \beta_2\frac{c}{r}\right)(1-\tau) = 0 \tag{74}$$

ii. When expansion occurs below the cap ($Q_{le}^{bar} < C$), the value for the equityholders is:

$$E^{e-bar}(Q,c) = \begin{cases} 0 & Q < Q_d^{e-bar}(c) \\ E_{11}^{e-bar}(c)Q^{\beta_1} + E_{12}^{e-bar}(c)Q^{\beta_2} & \\ \quad + \left(\dfrac{mQ}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) & Q_d^{e-bar}(c) \leqslant Q < Q_{le}^{bar} \\ E(Q, c_e^{bar})Q^{\beta_2} + D(Q, c_e^{bar}) & \\ \quad - D^{e-bar}(Q,c) - K_e & Q \geqslant Q_{le}^{bar} \end{cases}$$

where

$$E_{11}^{e-bar}(c) = \left(\left(E_2(c_e^{bar}) + D_2(c_e^{bar}) - E_{12}^{e-bar}(c) - D_2^{e-bar}(c)\right)(Q_{le}^{bar})^{\beta_2} + \frac{c_e^{bar} - c}{r}\tau - K_e\right)(Q_{le}^{bar})^{-\beta_1} \tag{75}$$

$$E_{12}^{e-bar}(c) = -\left(E_{11}^{e-bar}(c)(Q_d^{e-bar})^{\beta_1} + \left(\frac{mQ_d^{e-bar}}{r-\alpha} - \frac{c}{r}\right)(1-\tau)\right)(Q_d^{e-bar}(c))^{-\beta_2} \tag{76}$$

where the expansion threshold Q_{le}^{bar} is:

$$Q_{le}^{bar} = \left(\frac{\beta_1}{(\beta_1 - \beta_2)\left(E_2(c_e^{bar}) + D_2(c_e^{bar}) - E_{12}^{e-bar}(c) - D_2^{e-bar}(c)\right)}\left(K_e - \frac{c_e^{bar} - c}{r}\tau\right)\right)^{1/\beta_2} \tag{77}$$

and the default threshold is the solution to the following equation:

$$(\beta_1 - \beta_2)E_{11}^{e-bar}(c)(Q_d^{e-bar}(c))^{\beta_1} + \left((1-\beta_2)\frac{mQ_d^{e-bar}(c)}{r-\alpha} + \beta_2\frac{c}{r}\right)(1-\tau) = 0 \tag{78}$$

The value of the debt, considering that debtholders receive $(1-\phi)V_u^{e-bar}$ upon default, is:

$$D^{e-bar}(Q,c) = \frac{c}{r} + D_1^{e-bar}(c)Q^{\beta_1} + D_2^{e-bar}(c)Q^{\beta_2} \tag{79}$$

where

$$D_1^{e-bar}(c) = y(c)\left((Q_{le}^{bar})^{\beta_2}w(c) + (Q_d^{e-bar}(c))^{\beta_2}D_2(c_e^{bar})(Q_{le}^{bar})^{\beta_2}\right) \tag{80}$$

$$D_2^{e-bar}(c) = -y(c)\left((Q_{le}^{bar})^{\beta_1}w(c) + (Q_d^{e-bar}(c))^{\beta_1}D_2(c_e^{bar})(Q_{le}^{bar})^{\beta_2}\right) \tag{81}$$

and

$$y(c) = \frac{1}{(Q_d^{e-bar}(c))^{\beta_2}(Q_{le}^{bar})^{\beta_1} - (Q_d^{e-bar}(c))^{\beta_1}(Q_{le}^{bar})^{\beta_2}} \tag{82}$$

$$w(c) = -\left((1-\phi)\left(\left(G_{11}^{bar} + \Pi_{ue}^{bar}\left(\frac{1}{Q_{ue}^{bar}}\right)^{\beta_1}\right)(Q_d^{e-bar}(c))^{\beta_1} + \frac{Q_d^{e-bar}(c)(1-\tau)}{r-\alpha}\right) - \frac{c}{r}\right) \tag{83}$$

Shareholders optimally invest at the threshold Q_{le}^{bar} *and choose the the optimal coupon* c^{e-bar}*, that maximize the value of the idle firm:*

$$\{Q_{le}^{bar}, c^{e-bar}\} = \underset{\{Q_l, c\}}{\operatorname{argmax}}\left(E^{e-bar}(Q_l, c) + D^{e-bar}(Q_l, c) - K\right)\left(\frac{Q}{Q_l}\right)^{\beta_1} \tag{84}$$

4 Comparative statics

In this section we conduct a numerical comparative analysis to examine how the investment dynamics and financing policy are influenced by the presence of both the output cap and the upper reflecting barrier. The base-case parameter values are presented in Table 1:

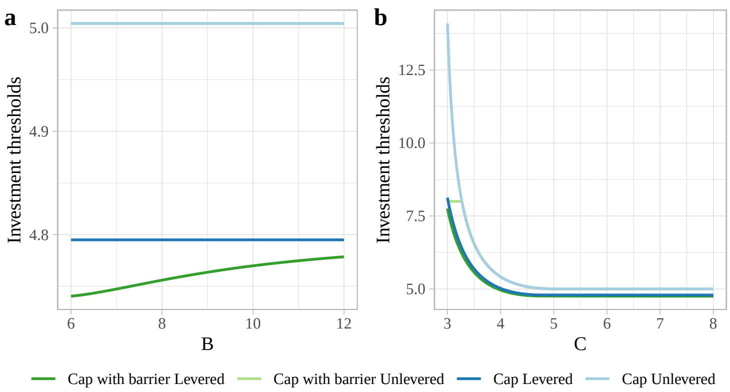
Parameter	Description	Value
$Q(0)$	Current level of Q	3
B	Barrier	8
C	Cap	6
σ	Volatility	0.2
r	Risk-free interest rate	0.04
α	Risk-neutral drift rate	0.015
τ	Corporate tax rate	0.15
K	Investment cost	60
ϕ	Bankruptcy cost	0.5

Table 1: The base-case parameter values.

4.1 Without the option to expand

The effect of leverage

Figure 4 depicts the effect of caps and barriers on the investment dynamics, both for the case of levered and unlevered firms.

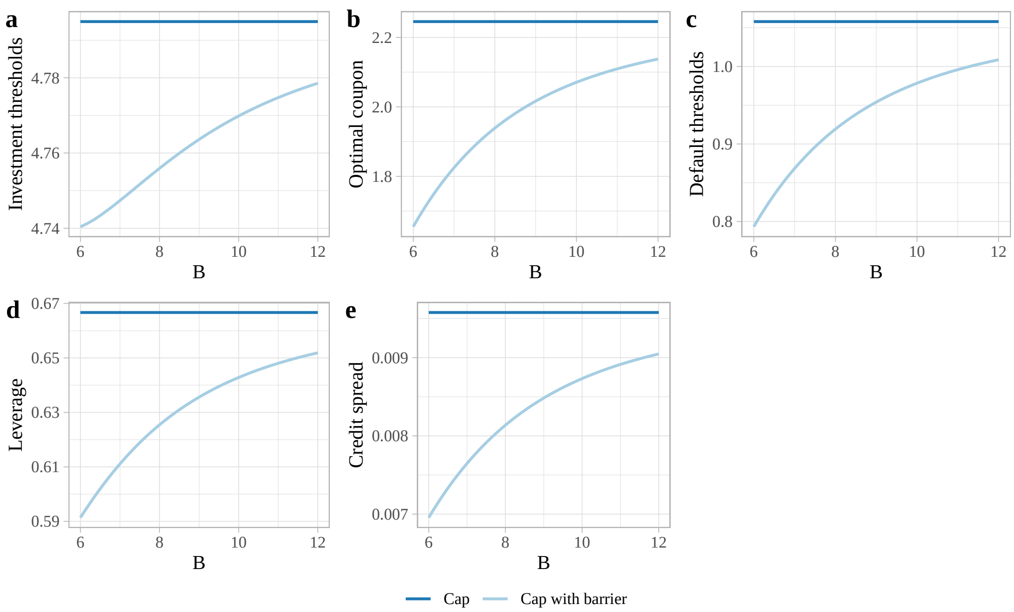


Parameter values as in Table 1.

Figure 4: The effect of leverage for different levels of the barrier and cap.

First, we see that leverage accelerates investment, as it lowers its threshold. As shown before, for the case of an unlevered firm, the reflecting barrier has no influence on investment dynamics. Conversely, for levered firms, the barrier demonstrates a modest capacity to accelerate investment (Figure 4a). Furthermore, significant decreases in caps deter investment, while the effect vanishes for large caps. The investment threshold is contingent on the presence or absence of the barrier. In the former case, the threshold consistently rises as the cap decreases. If demand is constrained by the barrier and the cap is sufficiently low, investment is deterred until the very last moment, i.e. until when demand reaches the barrier level (Figure 4b).

The effect of caps and barriers

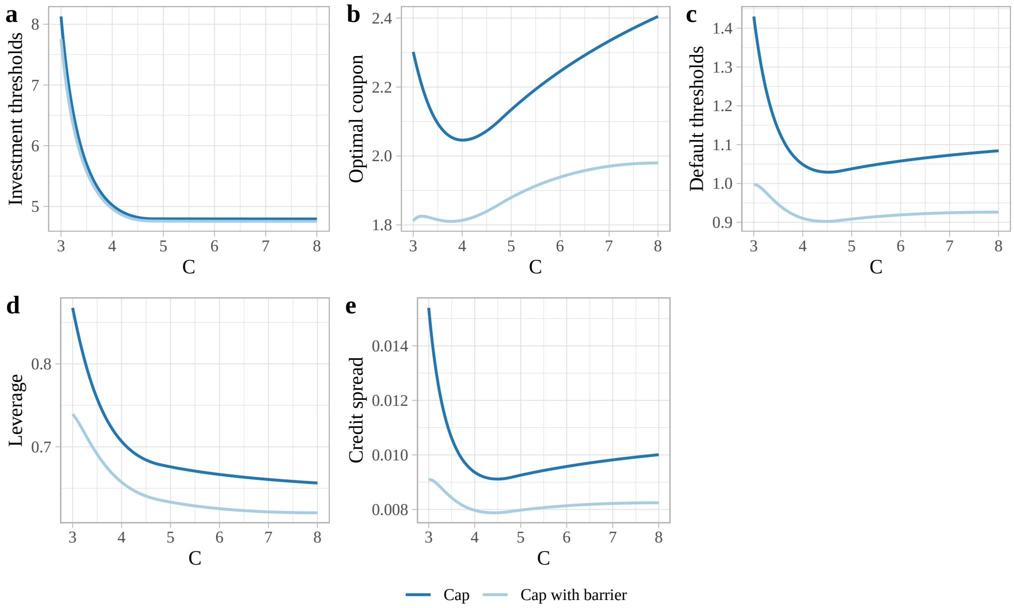


Parameter values as in Table 1.

Figure 5: The effect of the barrier (B)

Now, let us now focus on the implications of both a cap and a reflecting barrier on investment and financing policies. We begin by examining the effects of barriers, as illustrated in Figure 5. We find that a lower barrier results in a decrease in the level of optimal leverage, as shown in Figures 5b and 5d. Additionally, it serves as a deterrent against firm default, as indicated in Figure 5c, and contributes to a reduction in credit spreads, as depicted in Figure 5e. As previously mentioned, it is shown that the barrier has a limited effect on investment timing, as shown in Figure 5a by the small decrease of the investment threshold as the barrier is approaches the cap.

When considering the influence of caps (Figure 6), non-monotonic effects come into play on the optimal coupon, the timing for default and credit spreads (respectively, Figures 6b, c, and e). These non-monotonic effects are mitigated in the presence of a barrier. In any case, a lower cap leads to an increase in the firm's leverage as the level of the cap decreases (Figure 6d).



Parameter values as in Table 1.

Figure 6: The effect of the cap (C)

The effect of uncertainty

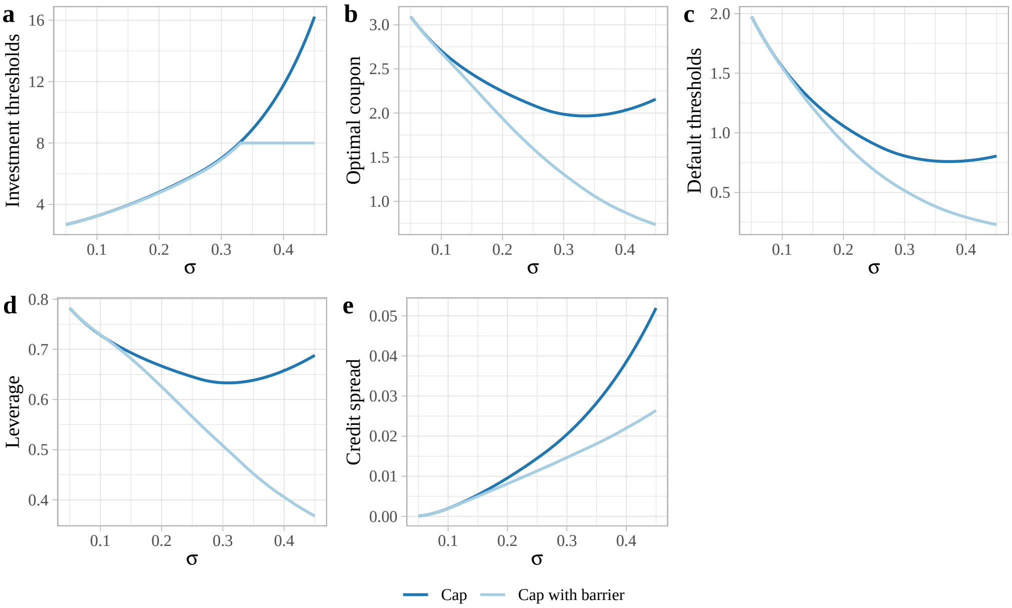
Figure 7 depicts the effect of uncertainty on investment and financing decisions. Firstly, in scenarios characterized by low uncertainty, the reflecting barrier has no effect on the investment timing. This suggests that under conditions of low uncertainty, barriers do not play a substantial role in influencing the decision-making for undertaking the project.

However, the effect of the upper reflecting barrier is more significant for higher levels of uncertainty, as the likelihood of delaying investment until the barrier is hit increases. Moreover, the barrier shows larger impacts on leverage, default, and credit spreads. In fact, under high uncertainty, the presence of the barrier reduces the optimal level of leverage, delays the timing for default, and increases credit spreads. Overall, the non-monotonic effects on leverage disappear with the barrier when compared to the cap-only case.

4.2 With the option to expand

Let us now consider the option to expand the initial project. In such a case, the firm is able to spend an investment cost of $K_e = 30$ to remove the cap, and therefore the barrier. Additionally, exercising this expansion option enables the firm to adjust the initial level of leverage.

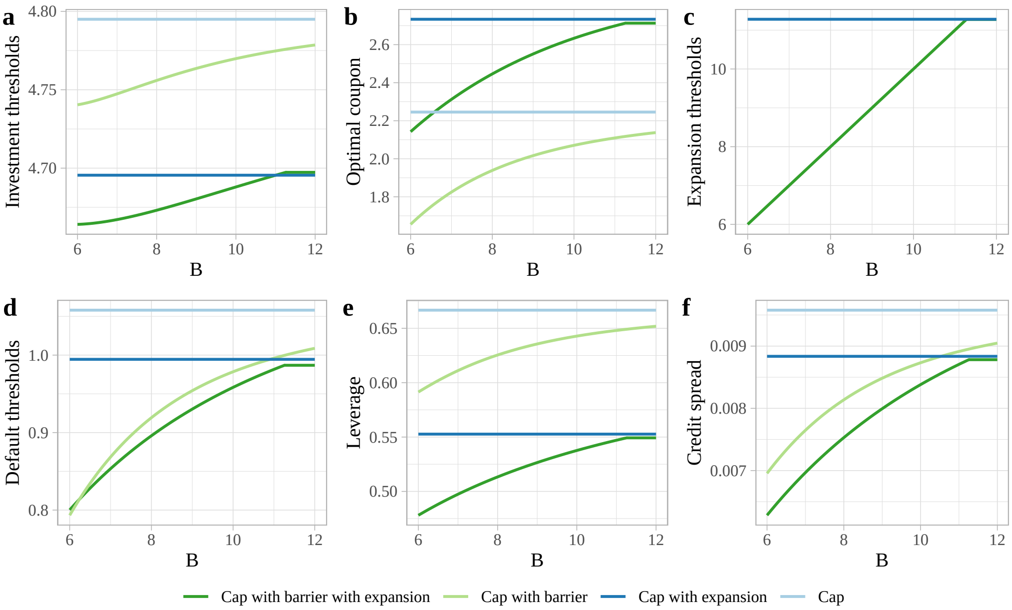
Figure 8 reveals that a higher barrier deters both the expansion (up to the barrier) and the initial investment. Even when the barrier has no effect on the expansion timing, it still affects the initial investment and leverage decisions. With the option to expand,



Parameter values as in Table 1.

Figure 7: The effect of uncertainty (σ)

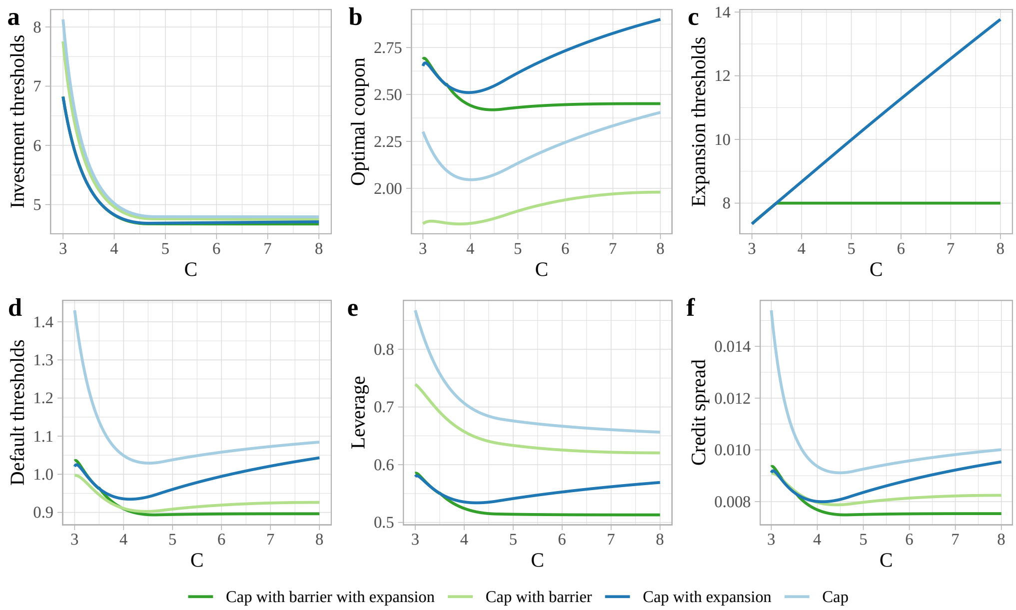
the initial investment accelerates, coupons increase, while the initial leverage ratios and credit spreads decrease.



Parameter values as in Table 1.

Figure 8: The effect of the barrier (B)

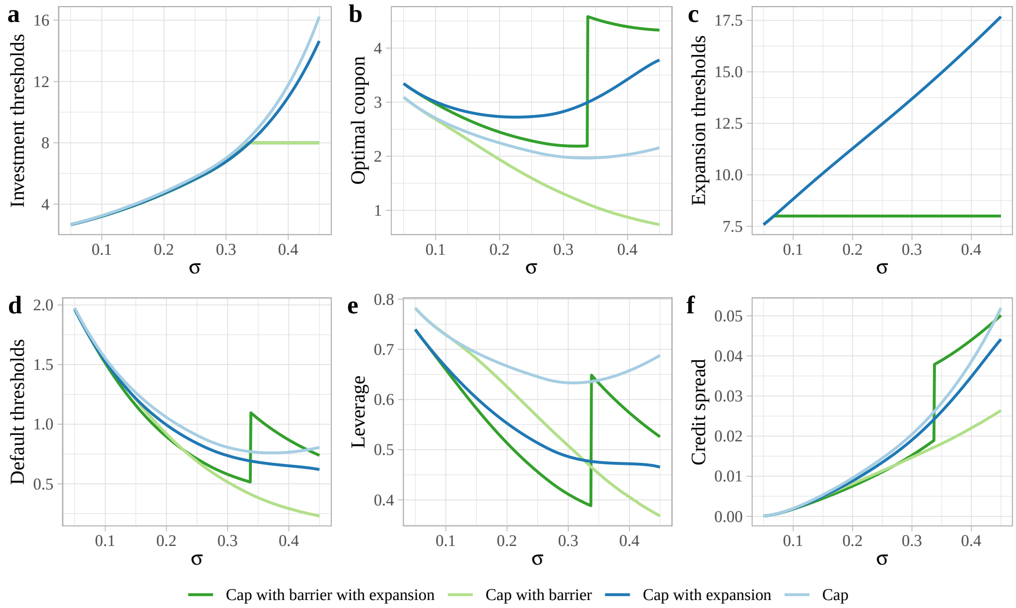
Figure 9 shows that a higher cap deters expansion (up to the barrier). Also, the effect of the expansion option remain. The softening of the non-monotonic effects of the barrier on the optimal coupon, the timing for default and credit spreads remain. However, without the barrier the effect on leverage ratio is now non-monotonic.



Parameter values as in Table 1.

Figure 9: The effect of the cap (C)

Finally, Figure 10 reveals that higher uncertainty deters both the initial and the expansion investment, unless when the thresholds approach the barrier. When the two investments take place simultaneously, a larger coupon is selected, instead of an initial smaller coupon that later is increased when expansion takes place. That explains the jumps observed in Figures 10b, 10d, 10e and 10f.



Parameter values as in Table 1.

Figure 10: The effect of uncertainty (σ)

5 Conclusions

This study sheds new light on the interactions between constraints on the supply side and demand side and the investment and financing decisions for firms, which seems particularly relevant in the context of infrastructure projects (e.g, airports).

Our findings reveal that while upper reflecting barriers prominently influence leverage decisions, their effect on investment timing is more subtle yet becomes pronounced under high uncertainty. Furthermore, the option to expand the project, removing all the constraints, induces early investment and modifies the financing policy, namely reducing leverage ratios and credit spreads.

This paper not only contributes to the existing literature by considering the joint effects of output caps, demand reflecting barriers, and investment and financing decisions, but also provides practical insights for industry practitioners and policymakers in managing and promoting infrastructure projects.

References

Adkins, R., Paxson, D., Pereira, P. J., and Rodrigues, A. (2019). Investment decisions with finite-lived collars. *Journal of Economic Dynamics and Control*, 103:185–204.

Dixit, A. (1991). Irreversible investment with price ceilings. *Journal of Political Economy*, 99(3):541–557.

Dixit, A. and Pindyck, R. (1994). *Investment Under Uncertainty*. Princeton University Press, New Jersey.

Dobbs, I. M. (2004). Intertemporal price cap regulation under uncertainty. *The Economic Journal*, 114(495):421–440.

Evans, L. and Guthrie, G. (2012). Price-cap regulation and the scale and timing of investment. *The RAND Journal of Economics*, 43(3):537–561.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, 49(4):1213–1252.

Nishihara, M. and Shibata, T. (2023). Optimal capital structure with earnings above a floor.

Rodrigues, A. (2023). Investment and leverage decisions under caps and floors. Available at SSRN: `https://ssrn.com/abstract=4498506`.

Sarkar, S. (2016). Consumer welfare and the strategic choice of price cap and leverage ratio. *The Quarterly Review of Economics and Finance*, 60:103–114.